# Noncommutative KKLMMT Model

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In the noncommutative space-time the fine tuning in KKLMMT model can be significantly released and a nice running of spectral index fitting the WMAP three year data can be achieved. The fitting results show that the noncommutative mass scale is roughly $5 \times 10^{14}$ Gev. The string mass scale is higher than the noncommutative scale unless the string coupling is smaller than $10^{-10}$.

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Inflation [1] economically solves the horizon problem, flatness problem and so on in the hot big bang model. It also predicts a nearly scale-invariant spectra of primordial scalar and tensor perturbations. A wide range of astronomical data sets are consistent with the predictions of the ΛCDM model [2]. The results of WMAP three data are given in [2]. For ΛCDM model, WMAP three year data only shows that the index of the power spectrum satisfies

$$n_s = 0.951^{+0.015}_{-0.019}. \tag{1}$$

A red power spectrum is favored at least at the level of 2 standard deviations. If there is running of the spectral index, the constraints on the spectral index and its running are

$$n_s = 1.21^{+0.13}_{-0.16}, \quad \alpha_s = \frac{dn_s}{d\ln k} = -0.102^{+0.050}_{-0.043}, \tag{2}$$

and the tensor-scalar ratio satisfies

$$r \leq 1.5 \quad (95\%\mathrm{CL}). \tag{3}$$

Even though allowing for a running spectral index slightly improves the fit to the WMAP data, the improvement in the fit can not provide strong evidence to require the running. Many inflation models have been proposed in the last decade. The precise observational data has been used to rule out some of them, see for example [2-5,15]. But many inflation models still survive. Here we need to keep in mind that how to construct a realistic inflation model in a fundamental theory is still an open question.

Brane inflation model [6-8], which is very appealing in having such a UV completion, is proposed in string theory after the introduction of the concept of D-branes therein. However there is an $\eta$ problem in brane inflation model [8] which says the distance between the brane and anti-brane should be larger than the size of the extra dimensional space, since the branes are too heavy. A more realistic model is the so called KKLMMT model [10,11], where warping effects are employed to make the brane lighter and thus solve the $\eta$ problem. However the authors in [15] pointed out that a stringent fine tuning is still needed in order that KKLMMT model can fit the WMAP three year data.

In this short note, we investigate KKLMMT model in the noncommutative space-time. We find that the noncommutative effects can accommodate the WMAP results with running of the spectral index and release the fine tuning in KKLMMT model.

Noncommutative geometry can naturally emerge in string theory. In [16] the commutator between space and time coordinates is not zero if there is an electric field on the brane, which says

$$[t,x] = i\theta = iM_{nc}^{-2}, \tag{4}$$

where $\theta = \frac{1}{E_{cr}}\frac{\tilde{E}}{1-\tilde{E}^2}$, $\tilde{E} = E/E_{cr}$ and $E_{cr} = 1/(2\pi\alpha')$ is the critical electric field. Beyond that value strings can materialize out of the vacuum, stretch to infinity and destabilize the vacuum [16,17]. This noncommutativity leads to an uncertainty relation between time and space, which was advocated as a generic property of string theory even when no electric field is present [18].

In a quantum theory, time coordinate labels the evolution of the system. We don't know how the time could fail to commute. Here we adopt the strategy proposed in [19] to explore how the space-time noncommutative effects quantitatively modify the evolution of the quantum fluctuations during the period of inflation. There are many discussions about the parameter space of the KKLMMT model, see for example [11,13,14]. In this short note, we parameterize KKLMMT model with potential

$$V = \frac{1}{2}\beta H^2\phi^2 + 2T_3h^4\left(1 - \frac{M^4}{\phi^4}\right), \tag{5}$$

here

$$M^4 = \frac{27}{32\pi^2}T_3h^4. \tag{6}$$

The inflation is governed by the effective D3-brane tension on the brane

$$\tilde{T_3} = T_3h^4 = \frac{M_{obs}^4}{(2\pi)^3 g_s}, \tag{7}$$

where $h$ is the warped factor in the throat and $M_{obs} = M_s h$ is the effective string scale on the brane [12]. The warped factor makes D3-brane lighter. The $\beta$ term comes from the Kahler potential, D-term and also interactions in the superpotential, and $H$ is the Hubble constant. Generally $\beta$ is of order unity [10], but to achieve slow roll, it has to be fine-tuned to be much less than one [11,12,15] and it seems quite unnaturally. On the other hand, in [20-22], the authors find the space-time noncommutative effects can accommodate a large enough running. See [23,24] for the recent progresses. This model was later extensively studied in [25]. Other models with a large running are discussed in [26,27].

The spacetime noncommutative effects are encoded in a new product among functions, namely the star product, replacing the usual algebra product. The evolution of the

background is homogeneous and the standard cosmological equations of the inflation will not change. The value of $\phi$, namely, $\phi_N$ at the number of e-folds equals $N$ before the end of inflation is

$$\phi_N^6 = 24NM_p^2M^4m(\beta), \tag{8}$$

where

$$m(\beta) = \frac{(1+2\beta)e^{2\beta N} - (1+\beta/3)}{2\beta(N+5/6)(1+\beta/3)}. \tag{9}$$

Now the slow roll parameter can be expressed as

$$\begin{aligned}
\epsilon_v &= \frac{1}{18}\left(\frac{\phi_N}{M_p}\right)^2\left(\beta + \frac{1}{2Nm(\beta)}\right)^2, \\
\eta_v &= \frac{\beta}{3} - \frac{5}{6}\frac{1}{Nm(\beta)}, \\
\xi_v &= \frac{5}{3Nm(\beta)}\left(\beta + \frac{1}{2Nm(\beta)}\right).
\end{aligned} \tag{10}$$

The amplitude of the primordial scalar power spectrum in noncommutative space-time takes the form, (see [22] in detail)

$$\Delta_{\mathcal{R}}^2 \simeq \frac{V/M_p^4}{24\pi^2\epsilon_v}(1+\mu)^{-4} = \left(\frac{2^5}{3\pi^4}\right)^{1/3}\left(\frac{T_3h^4}{M_p^4}\right)^{\frac{2}{3}} N^{\frac{5}{3}} f^{-\frac{4}{3}}(\beta)(1+\mu)^{-4}, \tag{11}$$

where

$$f(\beta) = m^{-5/4}(\beta)(1+2\beta Nm(\beta))^{\frac{3}{2}}, \tag{12}$$

$\mu = H^2k^2/(a^2M_{nc}^4)$ is the noncommutative parameter, $H$ and $V$ take the values when the fluctuation mode $k$ crosses the Hubble radius, $k$ is the comoving Fourier mode and $M_{nc}$ is the noncommutative mass scale. The factor $(1+\mu)^{-4}$ in eq (11) comes from the space-time noncommutative effects. Substituting eq. (6) and (8) into (11), we have

$$\frac{\phi_N}{M_p} = \left(\frac{27}{8}\right)^{\frac{1}{4}} m^{\frac{1}{6}}(\beta)f^{\frac{1}{3}}(\beta)N^{-\frac{1}{4}}\left(\Delta_{\mathcal{R}}^2\right)^{\frac{1}{4}}(1+\mu). \tag{13}$$

The normalization of the primordial scalar power spectrum is $\Delta_{\mathcal{R}}^2 \simeq 2\times10^{-9}$ for $N \sim 50$. Now we have

$$\epsilon_v = \frac{1}{4\sqrt{6N}}\left(\Delta_{\mathcal{R}}^2\right)^{\frac{1}{2}} m^{\frac{1}{3}}(\beta)f^{\frac{2}{3}}(\beta)\left(\beta + \frac{1}{2Nm(\beta)}\right)^2(1+\mu)^2. \tag{14}$$

The spectral index and its running are

$$
\begin{aligned}
n_s &= 1 - 6\epsilon_v + 2\eta_v + 16\epsilon_v\mu, \\
\alpha_s &= -24\epsilon_v^2 + 16\epsilon_v\eta_v - 2\xi_v - 32\epsilon_v\eta_v\mu,
\end{aligned}
\tag{15}
$$

with the tensor-scalar ratio

$$
r = 16\epsilon_v. \tag{16}
$$

The noncommutative effects encode in the parameter $\mu$ and these effects are negligible if $\epsilon_v$ is too small. For $\beta < 0.1$, the tensor perturbations can be negligible [15] and the noncommutative effects can not improve the fitting for the large running. When $\beta$ becomes large, $\epsilon_v$ becomes also large. For instance, the spectral index and its running for $\beta = 0.25$ are showed in fig. 1.

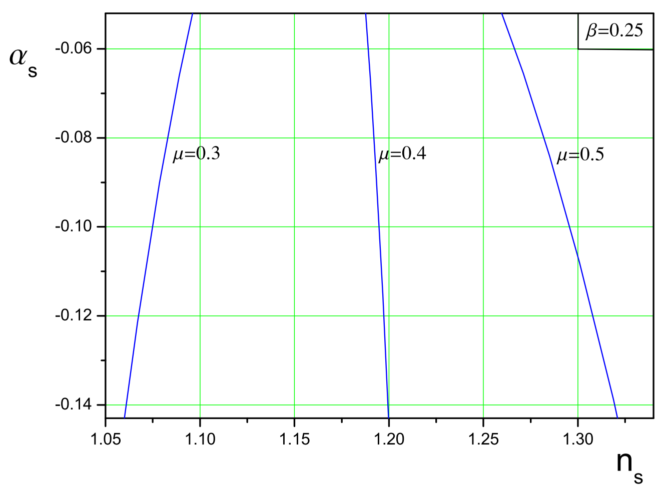


**Figure 1.** *The range for the spectral index and its running is allowed by WMAP three year data at the level of* $1\sigma$.

We also take $\beta = 0.25$ and $\beta = 0.3$ to fit three year results of WMAP respectively. The fitting results for eq. (2) are

$$
\begin{aligned}
N &= 47.7^{+1.6}_{-3.7}, \quad \mu = 0.414^{+0.241}_{-0.157}, \quad r = 1.12^{+0.31}_{-0.50}, \quad \text{for} \quad \beta = 0.25, \\
N &= 38.4^{+1.4}_{-2.8}, \quad \mu = 0.384^{+0.204}_{-0.148}, \quad r = 1.17^{+0.33}_{-0.51}, \quad \text{for} \quad \beta = 0.3,
\end{aligned}
\tag{17}
$$

at the level of one stand deviation. The allowed range for $\mu$ and the number of e-folds $N$ is showed in fig. 2 where the range of $\beta$ we scan is $0.22 \leq \beta \leq 0.31$.

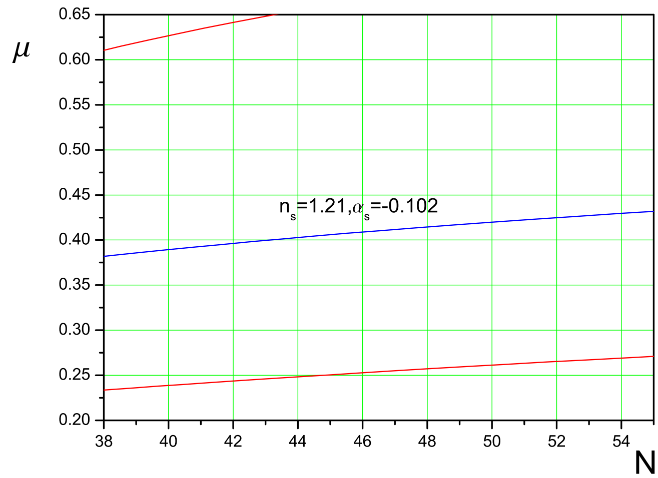


**Figure 2.** *Here the range of $\beta$ is $0.22 \le \beta \le 0.31$. The range between the two red lines is allowed by the WMAP three year data at the level of $1\sigma$. The blue line corresponds to $n_s = 1.21$ and $\alpha_s = -0.102$.*

The space-time noncommutative effects can improve KKLMMT model to nicely accommodate the spectral index and its running. Requiring $N \ge 47$ yields $\beta \le 0.25$.

Before the end of this note, we also want to work out the noncommutative mass scale. For $N = 50$, we read out that $\beta = 0.24$ and $\mu = 0.42$ corresponding to the blue line in fig. 2. Using eq. (11), we can decide the value of the effective D3-brane tension as

$$\tilde{T}_3 \sim 7 \times 10^{-8} M_p^4, \quad \text{or} \quad \frac{M_{obs}^4}{g_s} \sim 2 \times 10^{-5} M_p^4. \tag{18}$$

In order that the noncommutative effects become significant, the noncommutative mass scale is roughly the same as the Hubble constant during the period of inflation, namely $\mu$ is not quite smaller than one or $M_{nc} \sim H \sim \sqrt{\frac{2\tilde{T}_3}{3M_p^2}} \sim 2 \times 10^{-4} M_p \simeq 5 \times 10^{14}$ Gev. If $g_s \simeq 8 \times 10^{-11}$, $M_{nc} = M_{obs}$. We don't expect the string coupling is so small and thus the string mass scale is higher than the noncommutative mass scale.

In brane inflation, cosmic strings are possibly generated after the end of the inflation. Fitting cosmological constant plus cold dark matter plus strings to the CMB power spectrum provides an upper limit on the string tension with $GT \le 10^{-6}$ in [28] and recent constraint $GT \le 2.3 \times 10^{-7}$ in [29] (95% CL), where $T$ is the cosmic string tension. For D-string, $GT_D = (\frac{1}{32\pi g_s} \frac{\tilde{T}_3}{M_p^4})^{1/2} \sim 3 \times 10^{-5}/\sqrt{g_s}$. If cosmic D-sting was produced, $g_s \ge 10^4$.

For fundamental string, $GT_F = \frac{1}{8\pi}\left(\frac{M_{obs}}{M_p}\right)^2 \simeq 2\times 10^{-4}\sqrt{g_s}$. If cosmic fundamental string was generated, $g_s < 2\times 10^{-6}$.

To summarize, a nice running of the spectral index is obtained and the fine tuning for the parameter $\beta$ is significantly released in the noncommutative KKLMMT model. The noncommutative mass scale is roughly $5\times 10^{14}$ Gev which can be different from the string scale. If cosmic strings are produced after inflation, the constraint on the string coupling becomes stringent. We expect the future cosmological observations can provide stronger evidence to support a large amplitude of the tensor perturbations and a running of the spectral index for the primordial scalar power spectrum.

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